# Structural and Functional Robustness in Networks

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# Introduction

- Robustness is the ability of networks to **retain** their **functional** and **structural** properties in the presence of **variations**.
- Three models:
  1. Sandpile model.
  2. Road network structure using graph and GIS measures.
  3. A simple model of social organizational networks.
- In each case, we study how the relevant measures change as the structure of the network and the behavior of its nodes varies.


SAND

# Controlling Self-organizing Dynamics of Sandpile Model

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## Uncontrolled Model

- add one sand to a randomly chosen node
- if the node does not exceed capacity, cascade stops
- if it exceeds capacity, it topples and cascade begins: $h_i = 0$ and $h_j = h_j + 1$ with a dissipation probability of $\epsilon$

## Controlled Model

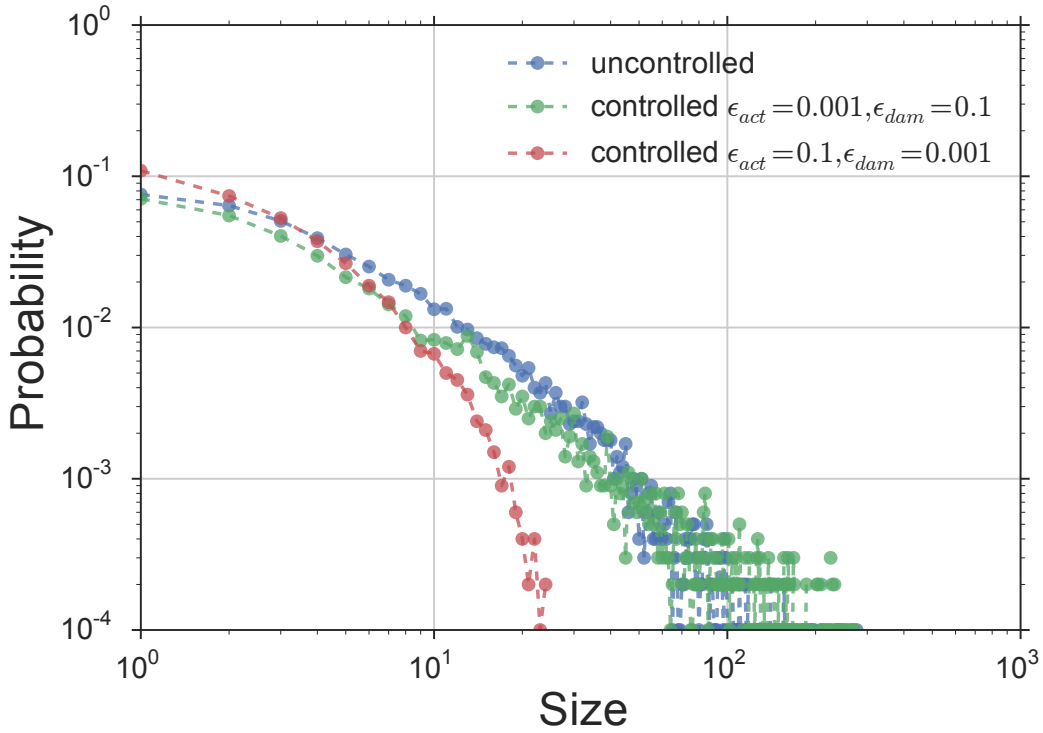
- do not allow the over-capacity node to immediately topple
- it damages with $\epsilon_{dam}$
- if it does not damage each of its sands dissipates with $\epsilon_{act}$
- if no damage and no active dissipation, toppling

size: number of toppling events
$n_{act}$: number of sands lost from active dissipation
$n_{dam}$: number of nodes damaged

# Results

Probability Generating Function (no damage)

$$F(x) = 1 - (1-\epsilon)\phi_{22}(1-\epsilon_{act})^3 + (1-\epsilon)\phi_{22}(1-\epsilon_{act})^3 x[F(x)]^2$$
$$G(x) = 1 - \psi_2 + \psi_2(1-(1-\epsilon_{act})^3) + \psi_2(1-\epsilon_{act})^3 x[F(x)]^3$$

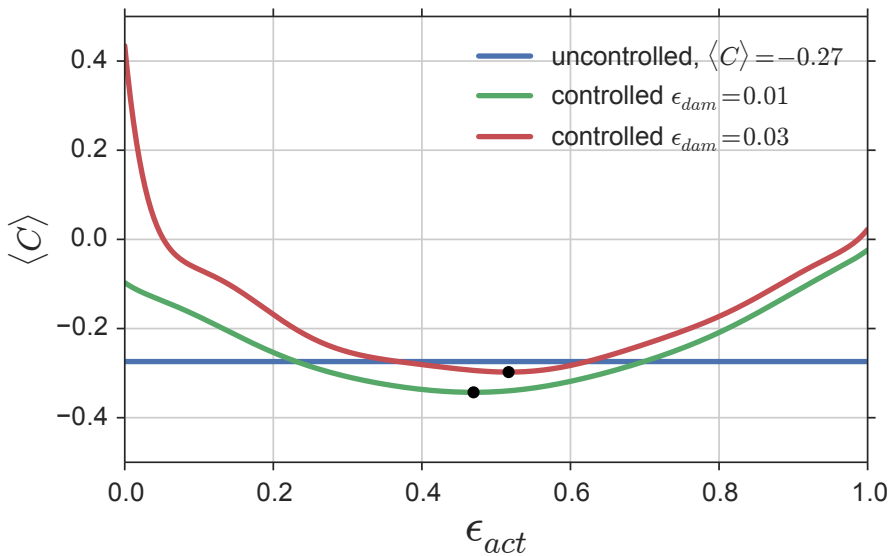



Probability Distribution of Cascade Size

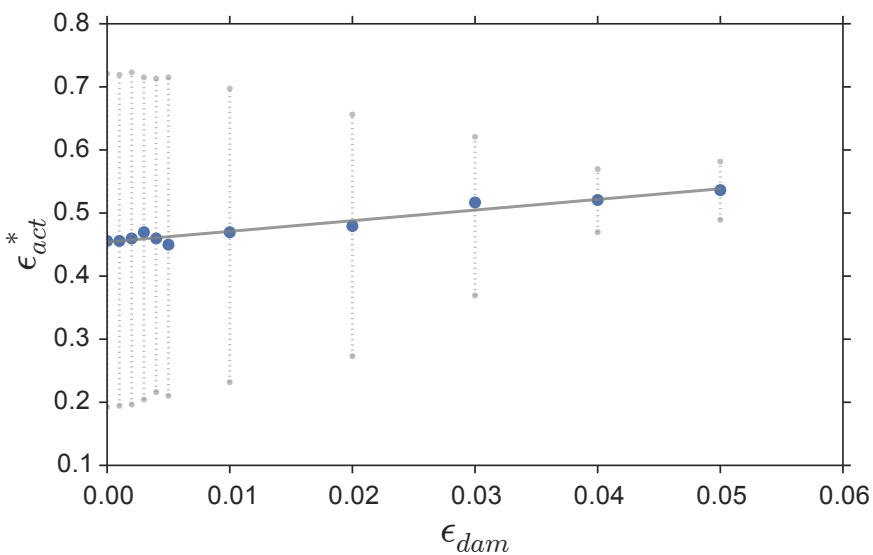
# Results

## Cost

$$C = C_{cascade} + C_{control}$$
$$= c[\text{size}]^{\alpha} + c_1 n_{act} + c_2 n_{dam}$$




Cost

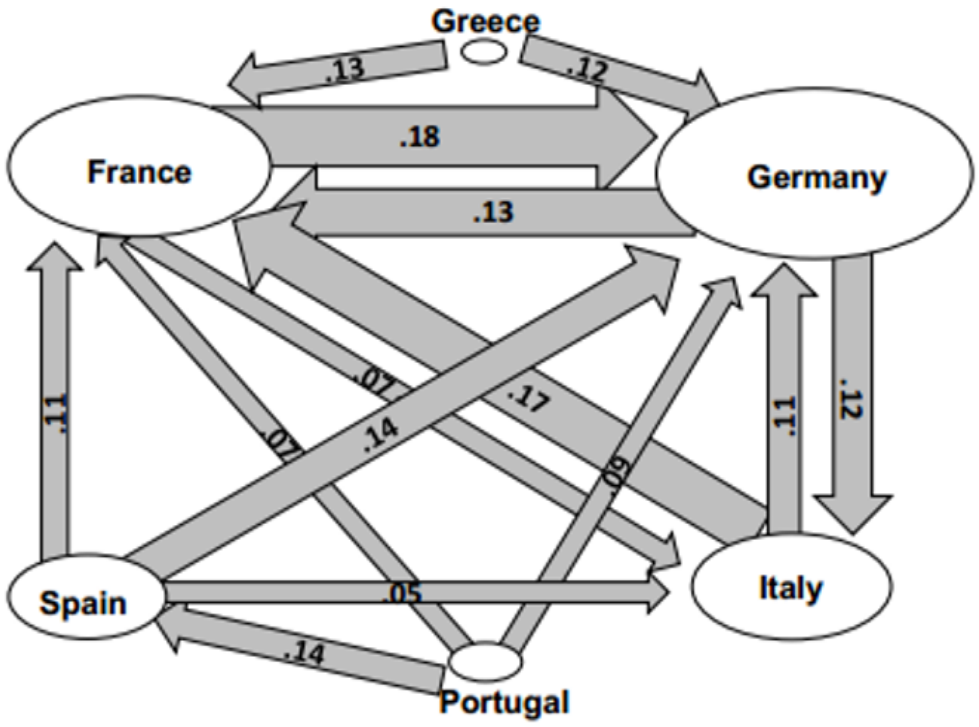



Optimal Control

What if we don't know the parameters?

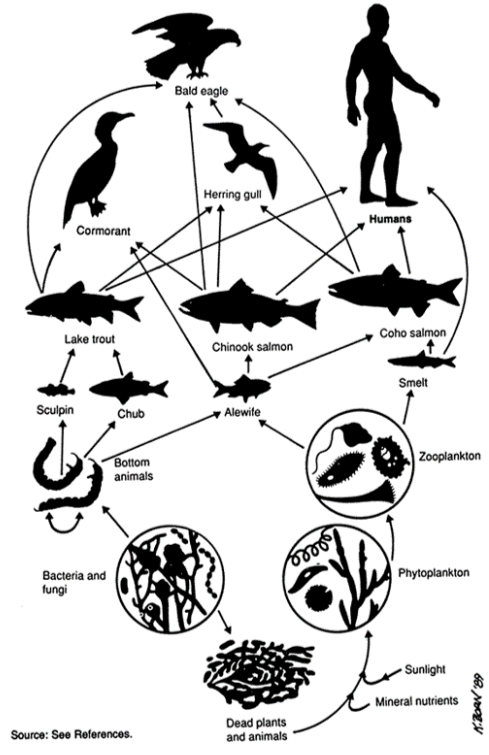
# Applications

## Cascading Debt Dependencies




arrows $\sim$ dependency size
oval size $\sim$ underlying asset value

## Extinction Cascades




loads estimated by metabolic rates on each trophic level

# Structural Robustness in Road Networks

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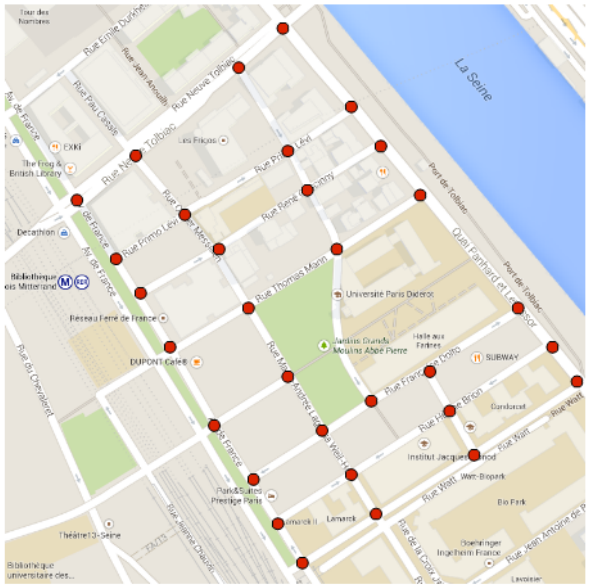
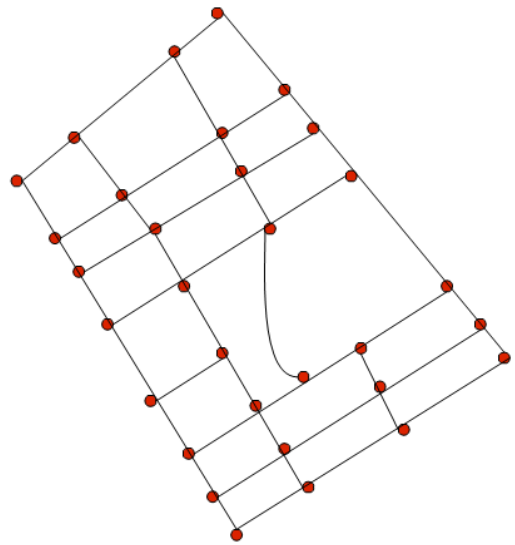
# Model

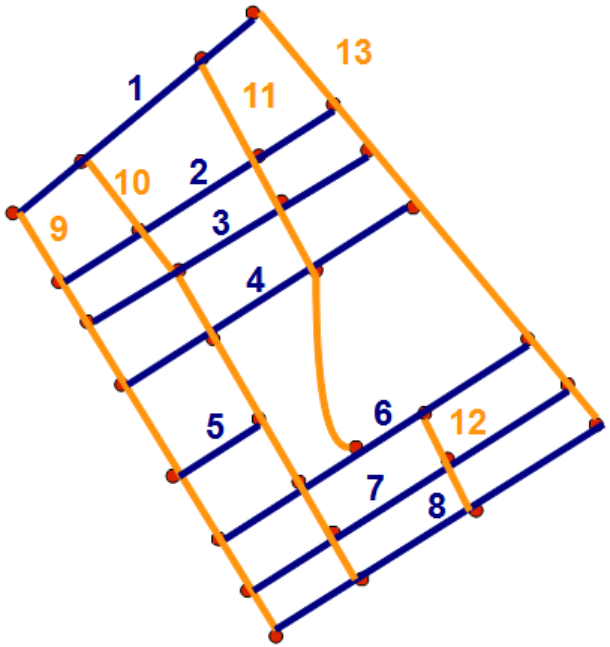
## Introduction to Road Network

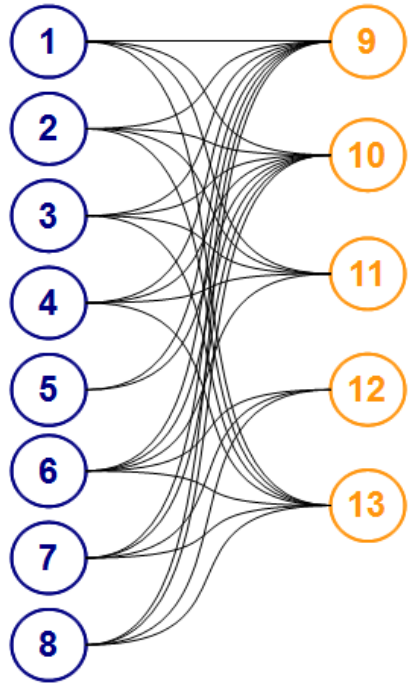


# Model

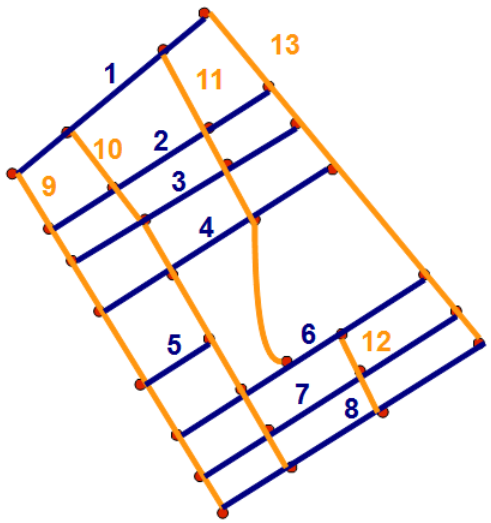
## Geographical vs Topological Representations



# Model

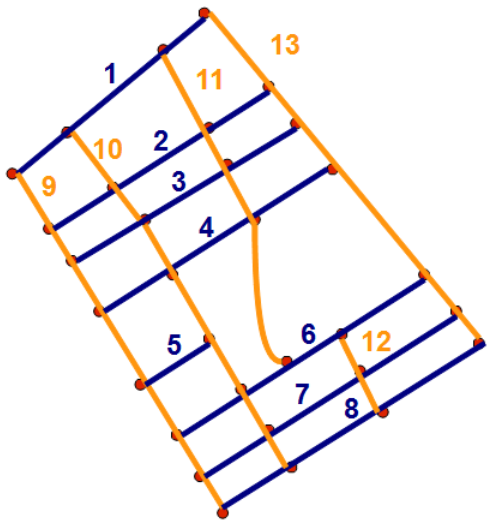
## Adjacency Matrix




| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 13 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

# Model

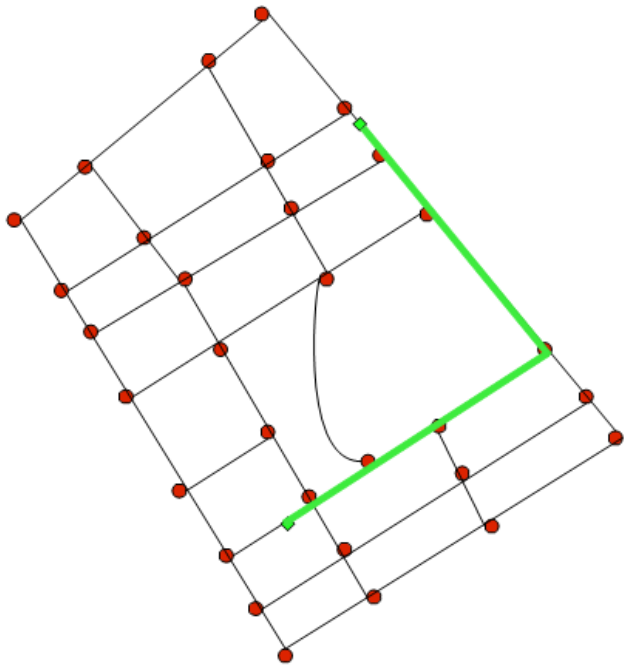
## Topological Distances Matrix



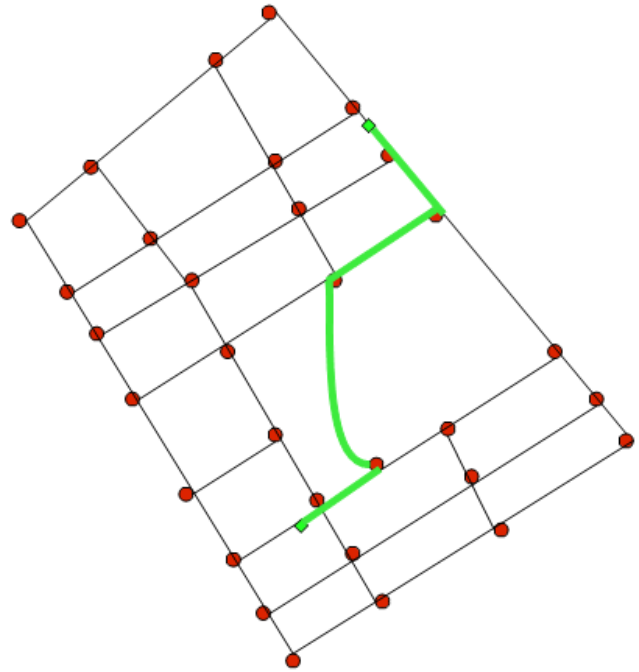

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 3 | 1 |
| 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 3 | 1 |
| 3 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 3 | 1 |
| 4 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 3 | 1 |
| 5 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 1 | 1 | 3 | 3 | 3 |
| 6 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 7 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 1 | 1 | 3 | 1 | 1 |
| 8 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 1 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 2 | 2 | 2 | 2 |
| 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 0 | 2 | 2 | 2 |
| 11 | 1 | 1 | 1 | 1 | 3 | 1 | 3 | 3 | 2 | 2 | 0 | 2 | 2 |
| 12 | 3 | 3 | 3 | 3 | 3 | 1 | 1 | 1 | 2 | 2 | 2 | 0 | 2 |
| 13 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 0 |

# Model

## Indicators



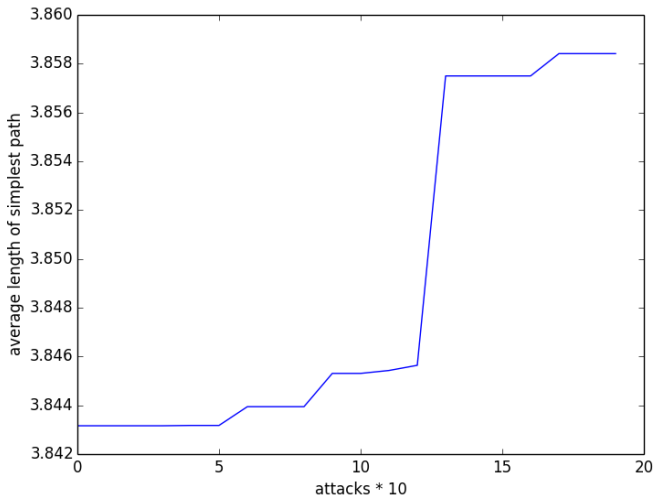
Simplest Path



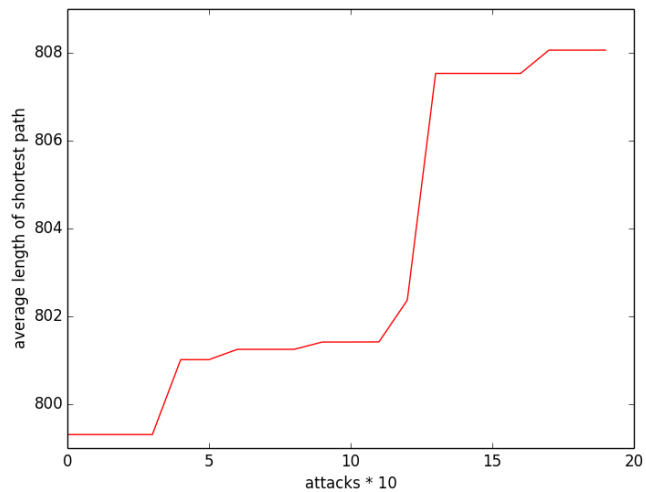
Shortest path

# First Results

## Network tolerance to failure



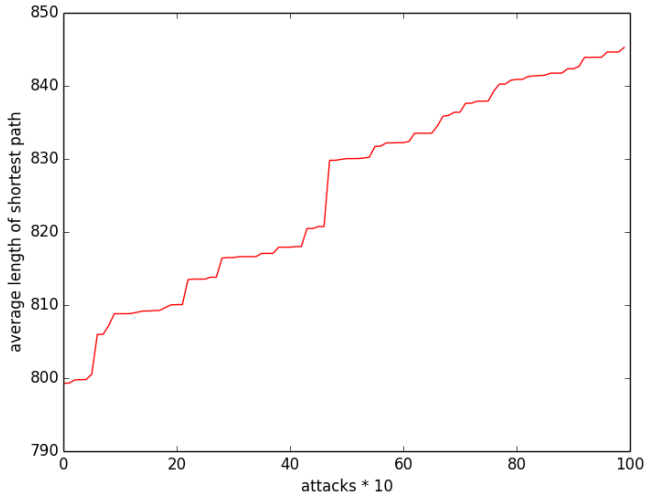

(k) Simplest Path : Removed Intersections



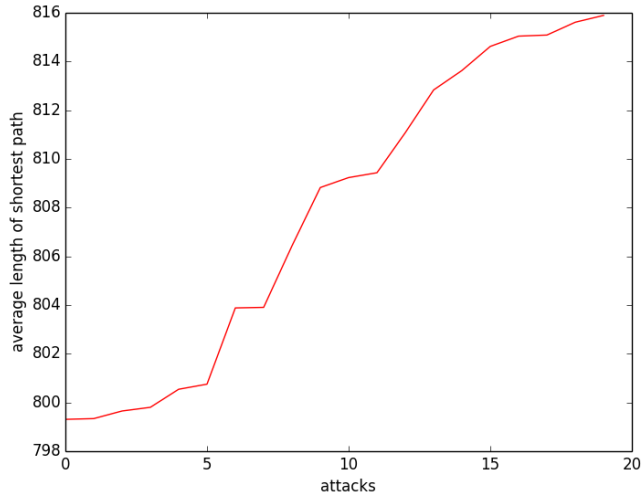

(l) Shortest Path : Removed Intersections

# First Results

## Network tolerance to failure

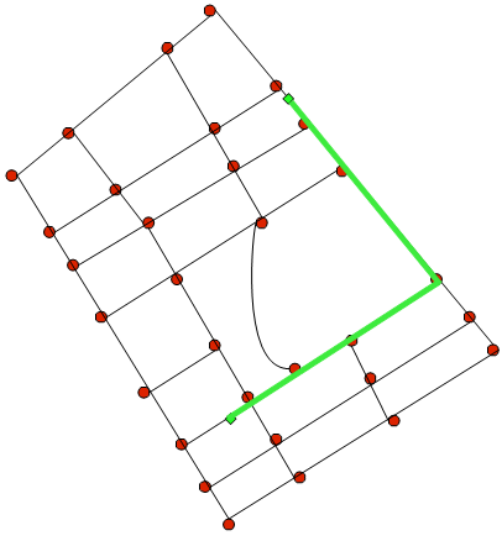



(m) Shortest Path : Intersections Weighted



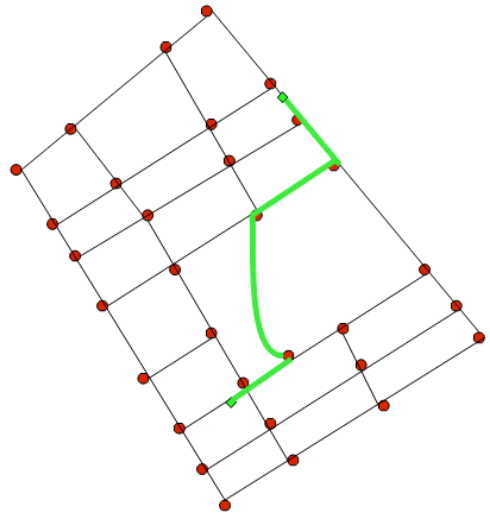

(n) Shortest Path : Roads Weighted

# Further Work



**Simplest Path**

→ Structurality



**Shortest path**

→ Betweenness

$$structurality(r_{ref}) = \sum_{r \in G} d_{simple}(r, r_{ref}) \times length(r)$$

$$betweenness(r_{ref}) = \sum_{r_{ref} \neq r_1 \neq r_2} \frac{\sigma_{r_1 r_2}(r_{ref})}{\sigma_{r_1 r_2}}$$

# Failure Tolerance in Social Organizational Networks

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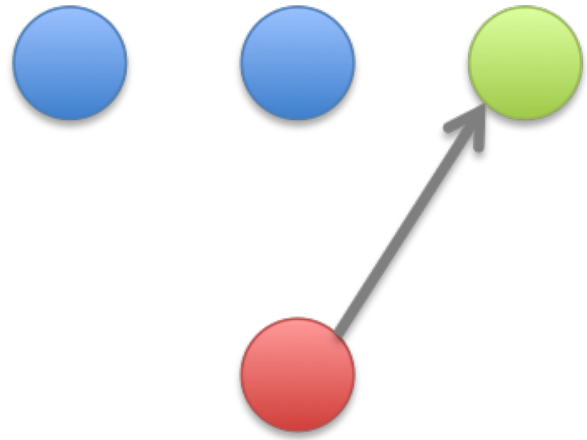
# Motivation

- Decision-making (Social) Agents
- Failure = Performance Shocks (Node Remains)
- Does failure tolerance vary with "relational cohesion?"
- How does this relationship affect the network?

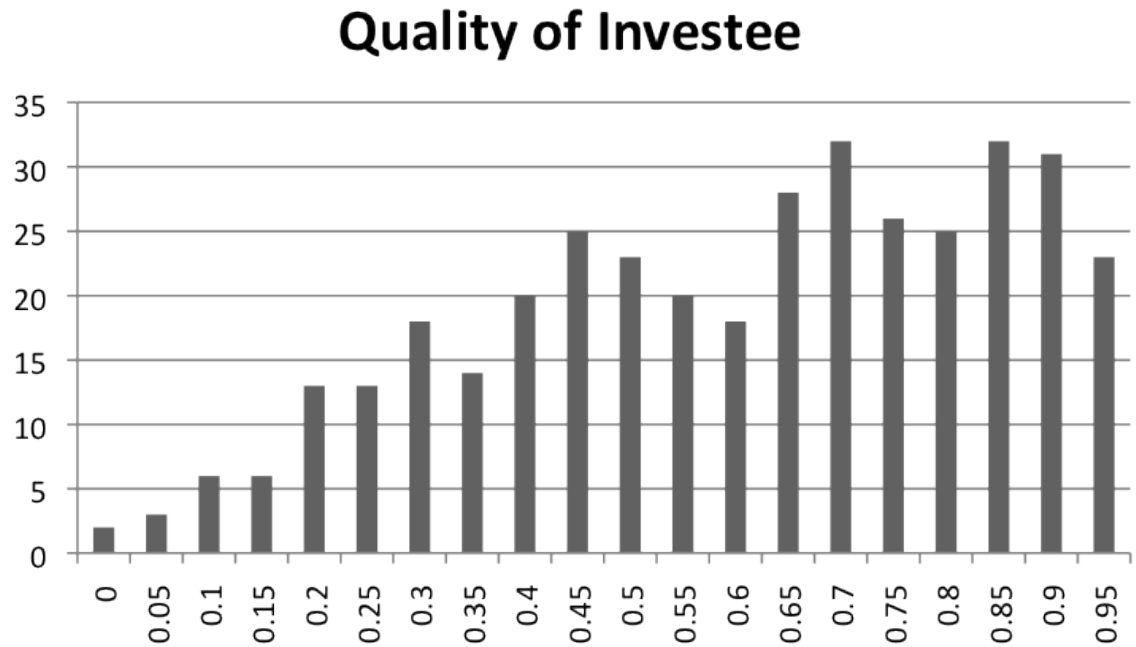
# Agent-Based Modeling of Failure Tolerance

- Dyadic Model
  - Performance Only (with Noise) ✓
  - Relational Cohesion - Hebbian (Memory) Model
    - Frequency Bias ✓
    - Multiplexity Bias
- Network Interactions



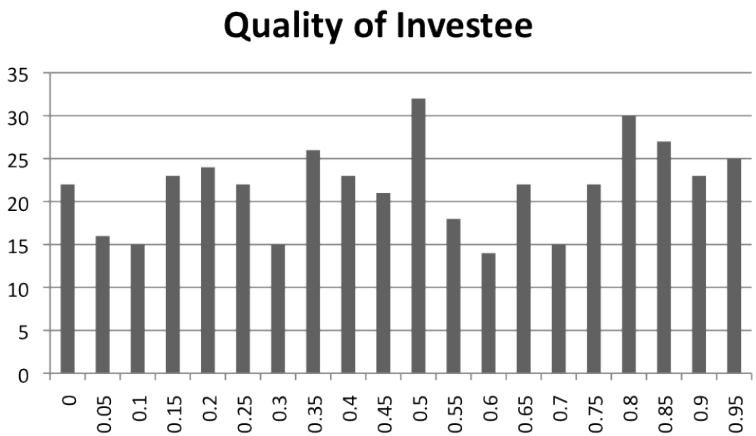
# Model 1 (Performance Alone)

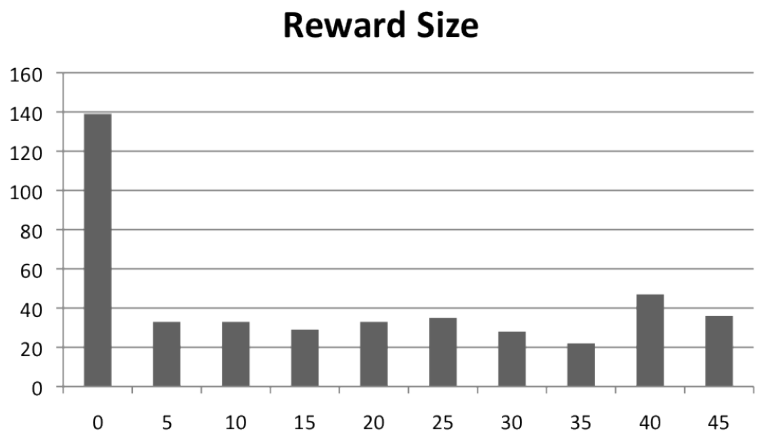
$$I_{AB_i} \propto Q_{B_i}$$



# Model 2 (Frequency Bias)

$$I_{AB_i}(t+1) \propto I_{AB_i}(t) + R(t) - D$$



# Future Work

- We conducted a preliminary study of **structural** and **functional** robustness in networks.
- We considered there model systems:
  1. Cascading failure and self-organized criticality.
  2. Topological robustness of road networks.
  3. Robustness in social organizational networks.
- Our next step is to find complete each study and look for **unifying themes** in these models.

# Acknowledgements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- SFI faculty
- Thank you for making CSSS 2014 possible.